

TEST NAME: 306 - Mathematical Sciences

LINEAR ALGEBRA AND VECTOR CALCULUS

1. Linear Algebra: Vector spaces, General properties of vector spaces, Vector subspaces, Algebra of subspaces, linear combination of vectors. Linear span, linear sum of two subspaces, Linear independence and dependence of vectors, Basis of vector space, Finite dimensional vector spaces, Dimension of a vector space, Dimension of a subspace. Linear transformations, linear operators, Range and null space of linear transformation, Rank and nullity of linear transformations, Linear transformations as vectors, Product of linear transformations, Invertible linear transformation. The adjoint or transpose of a linear transformation,

Sylvester's law of nullity, characteristic values and characteristic vectors, Cayley- Hamilton theorem, Diagonalizable operators. Inner product spaces, Euclidean and unitary spaces, Norm or length of a vector, Schwartz inequality, Orthogonality, Orthonormal set, complete orthonormal set, Gram - Schmidt orthogonalisation process.

2. Multiple integrals and Vector Calculus: Multiple integrals: Introduction, the concept of a plane, Curve, line integral- Sufficient condition for the existence of the integral. The area of a subset of R^2 , Calculation of double integrals, Jordan curve, Area, Change of the order of integration, Double integral as a limit, Change of variable in a double integration. Vector differentiation. Ordinary derivatives of vectors, Space curves, Continuity, Differentiability, Gradient, Divergence, Curl operators, Formulae involving these operators. Vector integration, Theorems of Gauss and Stokes, Green's theorem in plane and applications of these theorems.

ABSTRACT ALGEBRA & REAL ANALYSIS

3. GROUPS: Binary operations- Definitions and properties, Groups—Definition and elementary properties, Finite groups and group composition tables, Subgroups and cyclic subgroups. Permutations—Functions and permutations, groups of permutations, cycles and cyclic notation, even and odd permutations, The alternating groups. Cyclic groups - Elementary properties, The classification of cyclic groups, sub groups of finite cyclic groups. Isomorphism - Definition and elementary properties, Cayley's theorem, Groups of cosets, Applications, Normal subgroups - Factor groups, Criteria for the existence of a coset group, Inner automorphisms and normal subgroups, factor groups and simple groups, Homomorphism- Definition and elementary properties, The fundamental theorem of homomorphisms, applications.

4. RINGS: Definition and basic properties, Fields, Integral domains, divisors of zero and Cancellation laws, Integral domains, The characteristic of a ring, some non - commutative rings, Examples, Matrices over a field, The real quaternions, Homomorphism of Rings - Definition and elementary properties, Maximal and Prime ideals, Prime fields.

5. REAL NUMBERS: The Completeness Properties of R , Applications of the Supremum Property. Sequences and Series - Sequences and their limits, limit theorems, Monotonic Sequences, Sub-sequences and the Bolzano-Weierstrass theorem, The Cauchy's Criterion, Properly divergent sequences, Introduction to series, Absolute convergence, test for absolute convergence, test for non-absolute convergence. Continuous Functions-continuous functions, combinations of continuous functions, continuous functions on intervals, Uniform continuity.

6. DIFFERENTIATION AND INTEGRATION: The derivative, The mean value theorems,

L'Hospital Rule, Taylor's Theorem. Riemann integration - Riemann integral, Riemann integrable functions, Fundamental theorem.

DIFFERENTIAL EQUATIONS

7. Differential equations of first order and first degree : Linear differential equations; Differential equations reducible to linear form; Exact differential equations; Integrating factors; Change of variables; Simultaneous differential equations; Orthogonal trajectories.

8. Differential equations of the first order but not of the first degree: Equations solvable for p ; Equations solvable for y ; Equations solvable for x ; Equations that do not contain x (or y); Equations of the first degree in x and y - Clairaut's equation.

9. Higher order linear differential equations: Solution of homogeneous linear differential equations of order n with constant coefficients. Solution of the non-homogeneous linear differential equations with constant coefficients by means of polynomial operators. Method of undetermined coefficients; Method of variation of parameters; Linear differential equations with non-constant coefficients; The Cauchy-Euler equation.

10. System of linear differential equations: Solution of a system of linear equations with constant coefficients; An equivalent triangular system. Degenerate Case: $p_1(D) p_4(D) - p_2(D) p_3(D) = 0$.

SOLID GEOMETRY

11. The Plane: Equation of plane in terms of its intercepts on the axis, Equations of the plane through the given points, Length of the perpendicular from a given point to a given plane, Bisectors of angles between two planes, Combined equation of two planes, Orthogonal projection on a plane.

12. The Line: Equations of a line, Angle between a line and a plane, The condition that a given line may lie in a given plane, The condition that two given lines are coplanar, Number of arbitrary constants in the equations of a straight line. Sets of conditions which determine a line, The shortest distance between two lines. The length and equations of the line of shortest distance between two straight lines, Length of the perpendicular from a given point to a given line, Intersection of three planes, Triangular Prism.

13. The Sphere: Definition and equation of the sphere, Equation of the sphere through four given points, Plane sections of a sphere. Intersection of two spheres; Equation of a circle. Sphere through a given circle; Intersection of a sphere and a line. Power of a point; Tangent plane. Plane of contact. Polar plane, Pole of a plane, Conjugate points, Conjugate planes; Angle of intersection of two spheres. Conditions for two spheres to be orthogonal; Radical plane. Coaxial system of spheres; Simplified form of the equation of two spheres.

14. Cones, cylinders and conicoids : Definitions of a cone, vertex, guiding curve, generators. Equation of the cone with a given vertex and guiding curve. Enveloping cone of a sphere. Equations of cones with vertex at origin are homogenous. Condition that the general equation of the second degree should represent a cone. Condition that a cone may have three mutually perpendicular generators Intersection of a line and a quadric cone. Tangent lines and tangent plane at a point. Condition that a plane may touch a cone. Reciprocal cones. Intersection of two cones with a common vertex. Right circular cone. Equation of the right circular cone with a given vertex, axis and semi-vertical angle. Definition of a cylinder. Equation to the Cylinder whose generators intersect a given conic and are parallel to a given line, Enveloping cylinder of a sphere. The right circular cylinder. Equation of the right circular cylinder with a

given axis and radius. The general equation of the second degree and the various surfaces represented by it; Shapes of some surfaces. Nature of Ellipsoid. Nature of Hyperboloid of one sheet.
